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RELATION OF RIB SPACING TO STRESS IN WING PLANES.

by

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PREFACE - The change of spacing between ribs in a wing plane may entail, (1) change of air pressure, both in distribution and amount, (2) change of fiber stress in the fabric and the ribs, unless they be so altered in dimensions as to keep their stresses constant. The aerodynamic effect just stated should be studied in the wind tunnel, or still better in full-scale flight, with pressure collectors at many points on a median section of the wing. The stress relations to the fabric and the rib will be considered in some detail here.

FIBER STRESS IN FABRIC AND RIB.

STRESS IN FABRIC - It can be shown--see Appendix--that the tensile stress t per unit width in the fabric of a wing plane, at a point where the resultant air pressure on it is p, is approximately

$$t = pa^2/8c,$$

a being the distance between the ribs, and c the depth of bulge of the fabric midway between ribs at the locality in question. Hence keeping constant the air pressure p, and the shape of bulge, or c/a, the lineal tension t varies directly as a. If then the fabric thickness be directly as a, the fiber unit stress is the same for all practical rib spacings. And conversely if the fiber unit stress remain constant, with varying rib distances, the bulge shape must remain constant, assuming the initial tautness the same in all cases.

STRESS IN RIB - By well known mechanics, the unit stress in a simple beam, with either a concentrated or a uniform load, is of the form

$$S = A \frac{W L}{b d^2}$$

in which W is the load, L the length, b the width, d the depth. If the wing plane and its ribs remain geometrically similar, b and L increase directly as d, and W as d^2, since W varies as the area. The stress therefore takes the form

$$S = K \frac{d^2.d}{d.d^2} = \text{const.}$$

CONCLUSION - Considering therefore the wing plane simply as a static structure, and ignoring the question of aerodynamic efficiency, it appears that the unit stress in the rib and fabric will remain constant for constant p if the linear dimensions of both rib and fabric be increased alike, viz., if wing and fabric remain geometrically similar. Since the bulge as well as the structural dimensions remains geometrically similar, the whole distended plane remains so, and hence should have the same pressure distribution and efficiency. If therefore the Burgess rule of making the rib spacing always one-fifth of the chord of the plane be valid for any one plane, it must be valid for all others that are mechanically similar in structure and covering.

STRESS STRAIN RELATIONS IN WING PLANE FABRIC.

To determine whether a given fabric is suitable for a given rib spacing a stress-strain diagram is made for samples of the fabric. The relation is roughly

$$t = k e,$$

in which k is a constant, and e is the true strain, that is the stretch of a given length of sample divided by the unstretched length. If in the wing plane, a be the rib distance, r the radius of the bulge, the stretch of a is arc--less chord $= a^3/24\ r^2$, by a well known property of small circular arcs. Dividing this by the original chord length a gives the strain,

$$e = a^2/24\ r^2.$$

Instead of r we may write its value, t/p, from hydrostatics, and obtain a stress-strain relation for the bulge

$$24et^2 = a^2\ p^2.$$

This, for any fixed a, p, is a hyperbola in e, t, and may be plotted on a chart with the linear relation $t = k\ e$, as in Plate I.* Their intersection gives the actual tension generated by the air pressure p when the particular fabric represented by $t = k\ e$ covers the rib spacing a with initial tension zero. For any initial strain e_o the hyperbola would be $24\ (e-e_o)\ t = a^2\ p^2$, or the former curve shifted laterally by an amount e_o.

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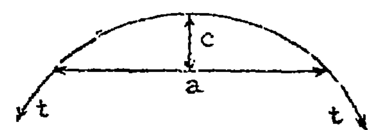
* Tech. Report, Brit. Adv. Com. Aeron. 1912-13, p. 232.

A P P E N D I X.

TO PROVE $t = pa^2/8c$ — If p, a, c, denote, as before, the unit resultant air pressure, rib spacing, and depth of bulge; and if r be the radius of curvature of the bulging fabric, assumed to be circular, as in the sketch; the following relations obviously obtain:



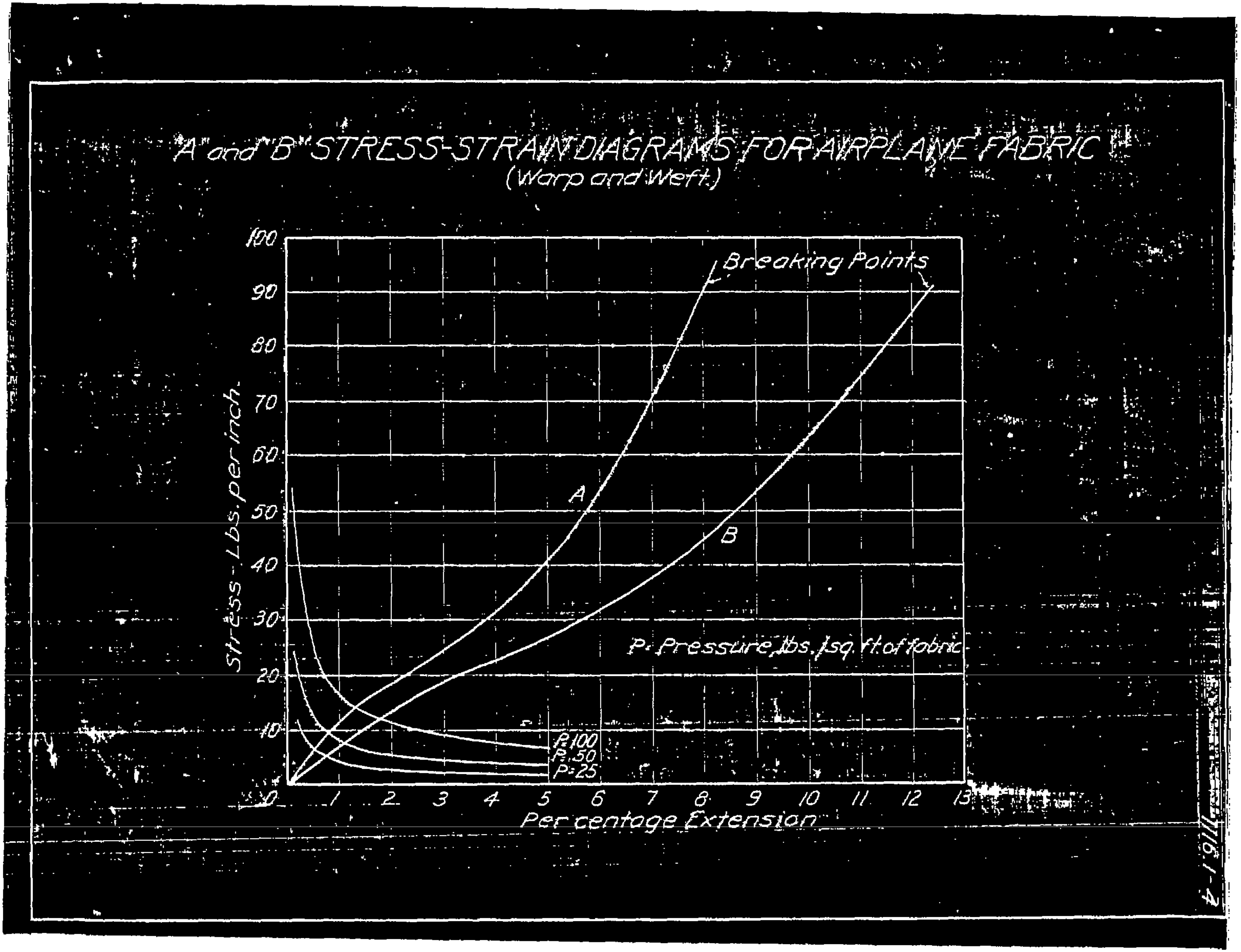
$$t = pr,$$
$$a^2 = 4c.(2r-c),$$
$$\therefore \quad t = \frac{pa^2}{8c}\left(1 + 4\frac{c^2}{a^2}\right)$$

The first of these comes from equating the up lift of the air to the down pull of the tension; the second from equating the square of the chord to four times the product of the segments of a normal diameter. The derived equation shows that if $c/a = 1/20$, $t = pa^2/8c$, accurately to one per cent. In taut fabrics c/a may equal 1/20, or thereabouts, under full load.

TENSION IN AIRPLANE FABRIC

$$t = \frac{p\ a^2}{144x8c}$$

Depth of Bulge c Inches	Tension t in Pounds per Lineal Inch for Various Air Pressures, p lbs. per sq. ft.							
	p = 5	p = 8	p = 10	p = 15	p = 20	p = 25	p = 50	p = 100
	For Rib Spacing a= 12"						t = p/8c	
0.1	6.25	10.0	12.50	18.75	25.0	31.25	62.50	125.0
0.2	3.12	5.0	6.25	9.37	12.5	15.62	31.25	62.5
0.3	2.08	3.33	4.16	6.25	8.33	10.41	20.83	41.66
0.4	1.56	2.5	3.12	4.68	6.25	7.81	15.62	31.25
0.5	1.25	2.0	2.50	3.75	5.00	6.25	12.50	25.0
	For Rib Spacing a = 13"						t = .1465 p/c	
0.1	7.32	11.72	14.65	22.0	29.30	36.60	73.25	146.5
0.2	3.66	5.86	7.32	11.0	14.65	18.30	36.62	73.25
0.3	2.44	3.91	4.88	7.33	9.76	12.20	24.41	48.8
0.4	1.83	2.93	3.66	5.5	7.32	9.15	18.31	36.6
0.5	1.46	2.34	2.93	4.4	5.86	7.32	14.65	29.3
	For Rib Spacing a = 14"						t = .1701 p/c	
0.1	8.50	13.61	17.01	25.51	34.02	42.52	85.05	170.1
0.2	4.25	6.80	8.50	12.75	17.01	21.26	42.52	85.0
0.3	2.83	4.53	5.67	8.50	11.34	14.17	28.35	56.7
0.4	2.12	3.40	4.25	6.37	8.50	10.63	21.26	42.52
0.5	1.70	2.72	3.40	5.10	6.80	8.50	17.01	34.0
	For Rib Spacing a = 15"						t = .1953 p/c	
0.1	9.76	15.62	19.53	29.29	39.06	48.82	97.65	195.3
0.2	4.88	7.81	9.76	14.64	19.53	24.41	48.82	97.6
0.3	3.25	5.20	6.51	9.76	13.02	16.27	32.55	65.1
0.4	2.44	3.90	4.88	7.32	9.76	12.20	24.41	48.8
0.5	1.95	3.12	3.90	5.85	7.81	9.76	19.53	39.0


"A" and "B" STRESS-STRAIN DIAGRAMS FOR AIRPLANE FABRIC
(Warp and Weft.)
Breaking Points
A
B
P = Pressure, lbs./sq. ft. of fabric
P=100
P=50
P=25
Stress - Lbs. per inch.
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
Percentage Extension
7116.1-4